Investigation of Microwave Propagation in High-Temperature Superconducting Waveguides

G. Yassin, *Member, IEEE*, G. Jung, V. Dikovsky, I. Barboy, M. Kambara, D. A. Cardwell, and S. Withington, *Member, IEEE*

***Abstract*—It is well known that the boundary conditions of the electromagnetic fields on the surface of a superconductor are influenced by the field penetration into the material. In a series of recent publications, it has been suggested that this effect substantially influences the wave propagation in high-temperature superconducting waveguides, to the extent that the mode order becomes different than that predicted for perfect conductor waveguides. In this paper, we present experimental investigation of this effect. We show that the effect of superconductivity on the wave propagation in waveguides is very small for temperatures well below the transition temperature and away from cutoff. We also discuss the behavior of the waveguide near cutoff and very close to the transition temperature.**

***Index Terms*—Superconducting waveguides.**

I. Introduction

PROPAGATION of electromagnetic waves in hollow tubes with normal conducting walls is usually described in terms of TE and TM modes. The boundary conditions of the electromagnetic fields on the perfect conductor surface restrict the cutoff wavenumbers to discrete values only. For example, the characteristic equation of a TE mode propagating in the z-direction in a cylindrical waveguide is given by

$$\frac{\partial H_z}{\partial r} = 0 \quad \Longrightarrow J_n'(k_c a) = 0 \tag{1}$$

where

- r radial coordinate;
- a waveguide radius;
- k_c cutoff wavenumber.

It is well known, however, that the boundary conditions on a high-T_c superconductor (HTS) are different from those on a perfect conductor as a result of the field penetration into the superconducting waveguide walls (Meissner Effect). In recent publications [1], [2], it has been suggested that alternative boundary conditions based on the Meissner Effect lead to a new characteristic equation which, for a cylindrical waveguide, may be written as

$$\frac{\partial H_z}{\partial r} = -\frac{1}{\lambda_L} H_z \quad \Longrightarrow (k_c a) J_n'(k_c a) + \frac{a}{\lambda_L} J_n(k_c a) = 0 \tag{2}$$

where λ_L is the London penetration depth. It is not difficult to show that for practical waveguide dimensions and for a temperature $T < T_c$ (where T_c is the transition temperature), equation (2) yields substantially different cutoff wavenumbers to those obtained from equation (1). For example, consider a cylindrical copper waveguide at Ku-band with internal radius $a \approx 8$ mm. The lowest order mode in this waveguide is the TE_{11} with a normalized cutoff wavenumber $(k_c a)_{11} \approx 1.84$. Assuming now that the waveguide was made of superconducting material with a typical penetration depth value $\lambda = 0.2\ \mu$m, equation (2) becomes $J_n(k_c a) \approx 0$. Therefore, the normalized wavenumber for the TE_{11} mode increases to $(k_c a)_{11} \approx 3.83$, while for the TE_{01} mode it is reduced to $(k_c a)_{01} \approx 2.405$. In other words, the above theory suggests that the fundamental mode is switched from TE_{11} to TE_{01}. Numerical solution of the transcendental equation (2) confirms this prediction (see [2, Table I]). Using the same theory, other workers [3], [4] analyzed propagation in a superconducting rectangular waveguide and came to the conclusion that the TE_{11} rather than the TE_{10} is the lowest order mode in the waveguide.

The analysis proposed above, if true, has substantial technological and theoretical implications. As a result we designed an experimental system to test its validity.

II. Experimental Results

Tests were carried out on a 50 mm long Y–Ba–Cu–O (YBCO) cylindrical waveguide which was fabricated at the IRC in Superconductivity, University of Cambridge, Cambridge, U.K. The waveguide consisted of six disks, each of 16.3 mm inner diameter and 32 mm outer diameter. Large grain Y–Ba–Cu–O samples were processed in batches of four in a purpose built thermal gradient furnace using a top seeded melt growth (TSMG) technique. This involved placing a small single crystal NdBCO seed on the top surface of each pressed pellet and heating the arrangement above the peritectic decomposition temperature of YBCO. The partially decomposed sample was then cooled below its solidification temperature and processed isothermally for 48 h to yield a single grain up to 50 mm in diameter. The samples were oxygenated in a separate process to enhance their superconducting properties. The disks were then machined flat and a cylindrical tube of internal diameter 16.3 mm was drilled ultrasonically at the center of each. The disks were finally re-oxygenated and the cylindrical waveguide was constructed by gluing the disks together with silver epoxy.

We first measured the transition temperature of each sample and found that it varied between 87–90 K. This ensured that

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G. Yassin and S. Withington are with Cavendish Laboratory, University of Cambridge, Cambridge, U.K.

G. Jung, V. Dikovsky and I. Barboy are with the Department of Physics, Ben-Gurion University of the Negev, Beersheva, Israel.

M. Kambara and D. A. Cardwell are with the IRC in Superconductivity, University of Cambridge, Cambridge, U.K.

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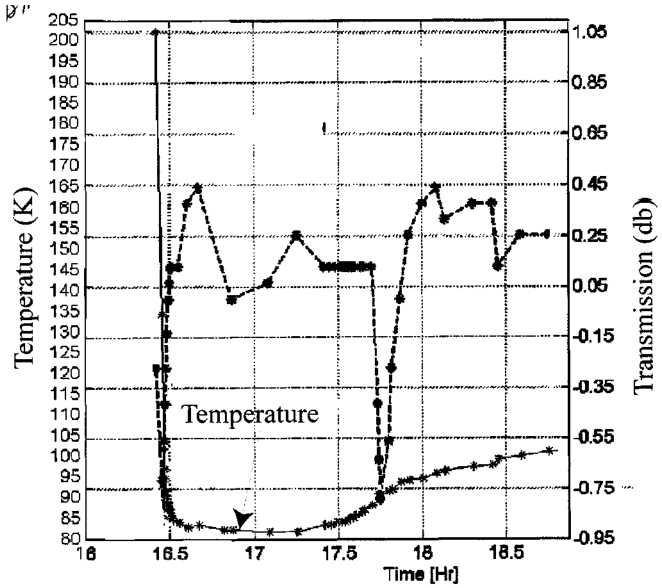

Fig. 1. Radiation pattern of a corrugated horn fed by a YBCO cylindrical waveguide which is either in the normal state (left side) or in the superconducting state (right side). The solid line is the computed pattern for a corrugated horn fed by perfect conductor waveguide.

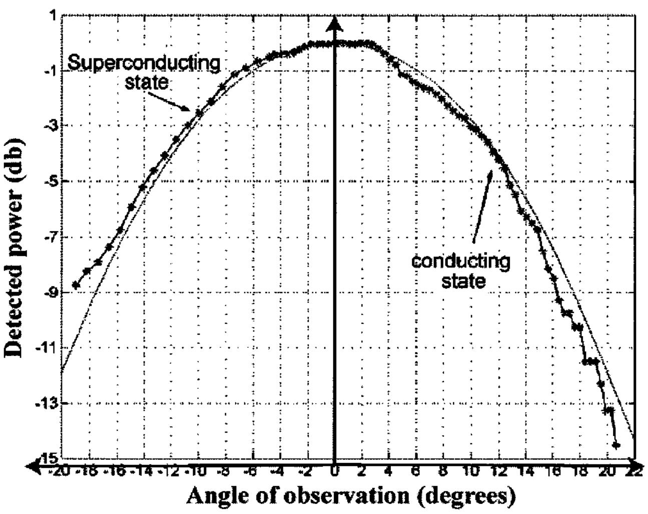

Fig. 2. Power transmission through the YBCO waveguide as a function of temperature. The dips in the transmission curve coincide to the phase transition from the normal to the superconducting state and visa versa.

all the disks made the transition from the normal to the superconducting state when they were cooled to the liquid Nitrogen temperature (77 K). To investigate the dependence of the cutoff frequency on the material state, we constructed a Ku-band transmission test system. A 50 mW Gunn diode oscillator excited the TE_{10} at 15.3 GHz in a rectangular waveguide. A commercial rectangular-to-circular transition was then used to launch the TE_{11} mode into the YBCO test section. This mode was then fed to a conical corrugated horn of 15° semiflare angle to excite the HE_{11} hybrid mode. In this way, we were able to explore the field distribution in the YBCO waveguide by measuring the radiation pattern of the corrugated horn, when the YBCO was either in the normal or superconducting state.

We started our radiation pattern tests with the YBCO section at room temperature by measuring the power radiated by the horn, in the far field, as a function of the azimuthal angle ϕ. The result is shown in Fig. 1 and compared with the far-field pattern computed using Kirchhoff's integration of the field distribution across the horn aperture (which was assumed to be equal to that of a balanced HE_{11} distribution multiplied by the quadratic phase error at the aperture of the horn) [5]. The measured and computed results agreed well, as can be seen from Fig. 1. The small discrepancies near the on-line direction ($\phi = 0$) of the pattern were caused primarily by standing waves between the transmitter and receiver flanges. The quality of the measured pattern, however, was sufficiently good to confirm that the pattern was indeed radiated by the HE_{11} mode. Next, we cooled the YBCO section by filling the cryostat with LN2. The temperature of the waveguide was monitored by three thermocouples connected to the ends and middle of the waveguide and the phase transition was monitored by measuring the inductance of three coils which were wound around the middle and extreme disks. After observing the three disks making transition to the superconducting state and the temperature reaching $\approx$80 K we took a radiation pattern without altering the detector position relative to the corrugated horn. There was only a tiny change to the transmitted power and the measured pattern was similar to the one obtained when the YBCO was in the normal state, as shown in Fig. 2. We repeated the same experiment by removing the corrugated horn and connecting the YBCO section output directly to the detector via a circular to rectangular transition. Again, only a tiny change to the transmitted power was observed as a result of the phase transition. Finally, we investigated the transmission as a function of temperature of the YBCO waveguide.

This was done by adjusting the detector position to be on-line with the radiating horn (maximum transmission) and then gradually cooling the YBCO waveguide. A selected result is shown in Fig. 2, which indicates that there was a sharp attenuation in transmission ($\approx$10 db/meter) as the YBCO waveguide makes the transition from the normal to the superconducting state and visa versa.

III. Discussion

The results described in the previous section demonstrate that well below the transition temperature, the superconducting YBCO waveguide was able to transmit the TE_{11} mode without significant attenuation. Neither the transmitted power nor the shape of the radiation pattern were altered when the YBCO waveguide became superconducting. This experimental result contradicts eqn. (2), which predicts that the cutoff frequency of the TE_{11} mode in a YBCO HTS of this diameter is more than $\approx$22 GHz (which is much higher than the frequency of our transmitter). In addition, it is easy to see that the TE_{01} mode could not possibly have been excited in the YBCO waveguide, as eqn. (2) predicts, since the power carried by this mode would have been heavily attenuated in the horn throat, causing substantial reduction in the transmitted power. We, therefore, conclude that the mode order in the superconducting YBCO remained the same as in a perfect conductor waveguide, which is determined by eqn. (1).

Close inspection of equation (2) explains its incorrect predictions. The equation $((\partial H_z)/(\partial r)) = -(1/\lambda)H_z$ describes the exponential decay of the tangential magnetic field inside the superconductor. Replacing H_z in this equation by the field of the cylindrical waveguide at the boundary [as was done in eqn (2)] assumes that both the tangential field and its normal derivative $((\partial H_z)/(\partial r))$ are continuous across the boundary defined by the waveguide inner surface. This assumption is incorrec, however, since the normal derivative $((\partial H_z)/(\partial r))$ is, in general, singular at the boundary.

To see how the parameters of an HTS waveguide really behave, we use the usual method of first calculating the normal conductor quantities and then replacing the normal conductivity σ_n with the complex conductivity $\sigma_c = \sigma_1 + i\sigma_2$. The quantities σ_1, σ_2 can then be calculated from the Mattis–Bardeen equations or the two-fluid model. Consider for simplicity the case of the TM_{01} mode. Well below cutoff, it is easy to show that a first order estimate of the change in the cutoff wavenumber as a result of the finite conductivity is given by [6]

$$\frac{\Delta k_c}{k_\circ} \approx (\chi_{01}a)^{-1}\frac{\mathrm{X}_s}{Z_\circ}$$

where

- $\mathrm{X_s}$ surface reactance;
- $Z_\circ$ impedance of free space;
- $k_\circ$ free space wavenumber;
- χ_{01} cutoff wavenumber of the TM_{01} mode.

This suggests that the fractional change in the cutoff wavenumber is proportional to the surface reactance. Since well above the waveguide cutoff and for $T \ll T_c$, $((\Delta k_c)/(k_\circ)) \ll (\chi_{01}a)$, this fractional change is very small. Alternatively, we use the fact that the surface reactance is given by $\mathrm{X_s} = \omega\mu_\circ\lambda_L$ and find that the fractional change is simply proportional to $((\lambda_L)/a)$, which is, in general, very small. This result is not at all surprising and indicates that well below the transition temperature, the energy stored inside the waveguide walls is much smaller than the energy propagating through the waveguide volume.

An alternative approximation based on a perturbation method by Jackson [7] yields the expression

$$\frac{\beta^2 - \beta_\circ^2}{\beta_\circ^2} = 2i\left(\frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}\right)(\beta_\circ a)^{-1})\frac{Z_s}{Z_\circ} \qquad (4)$$

where β and $\beta_\circ$ are respectively the propagation constants for the superconducting and perfect conductor states and Z_s is the surface impedance. Notice that the change in the complex propagation constant of the waveguide is determined by the surface impedance, which agrees with (3) and our experimental observations.

Finally, we discuss the behavior of the YBCO waveguide at temperatures $T \simeq T_c$, as shown in Fig. 2. We clearly see that the attenuation of the system has increased by ≈10 db/m as the system makes the transition from the normal to the superconducting state and visa versa. We attribute this effect to the expected sharp increase in penetration depth near the transition temperature. Such an increase is predicted theoretically and has also been observed experimentally in various HTS systems. We would like to emphasize, however, that the result shown in Fig. 2 should be considered preliminary and needs further investigation using a waveguide with uniform transition temperature and a system with more accurate temperature control.

IV. Conclusions

We have demonstrated experimentally that the propagation properties of the TE_{11} mode in a cylindrical YBCO HTS waveguide are similar to those in a perfect conductor. We have derived first order approximation expression to support this observation. Our investigation, however, revealed that a different behavior may be expected near the transition temperature and near cutoff. In the first case, the penetration depth increases rapidly as we approach T_c, which may cause significant and measurable change in the propagation constant. In the second case (near cutoff), the waveguide behaves as a cylindrical cavity and a small change in the propagation constant becomes measurable as a result of a phase switch. In either case, accurate measurement of this change would lead to a new method of determining the penetration depth in HTS materials. Experimental verification of this effect is in progress.

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